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# BRIEF COMMUNICATION

# AN EXTENDED CHARNOCK ESTIMATE OF INTERFACIAL STRESS IN STRATIFIED TWO-PHASE FLOWS

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**Abstract**—The existing theory of interfacial shear and roughness in fully-developed flow is generalized with the aid of the Colebrook–White formula to encompass smooth, transitional and rough interfaces, and an empirical correction improves the agreement with experiment near regime transition.

## INTRODUCTION

The stratified two-phase regime is one which arises frequently in the process and power-generation industries, among others, and important topical examples in nuclear reactors concern several types of postulated loss-of-coolant accidents in light-water reactors. A fundamentally significant aspect of such flows is the behaviour of the liquid/gas interface, insofar as it can dominate interphase and overall transport of momentum, heat and mass, and in view of the separated state of the two phases this regime has been amenable to quite extensive scrutiny from both the experimental and theoretical viewpoints.

Helpful surveys of the literature on turbulent exchange at solid surfaces as well as fluid interfaces have been provided by Wengefeld (1978) and Jensen & Yuen (1982). In a review of the current literature, Sinai (1986) noted that extensive studies of interfacial phenomena appear to have been carried out by workers in oceanography and civil engineering. Most of the material in the more complex field of internal two-phase flow has not assigned particular significance to the gas/liquid interface and has relied on several worrying empiricisms in predictive methods. Of particular concern is the use of an open-channel mean hydraulic diameter for the liquid phase and a standard smooth-wall friction factor at the interface [see, for example, the discussions in Agrawal *et al.* (1973), Taitel & Dukler (1976) and Butterworth & Hewitt (1979)], since there is ample evidence that under many conditions the interface is rough and is associated with relatively large interfacial shear (e.g. Hanratty & Engen 1957; Akai *et al.* 1981). It is for these reasons that Sinai (1983a, b) has recently investigated a technique for predicting the interfacial shear and roughness, based on Charnock's (1955) hypothesis. That hypothesis has its critics (Kraus 1972) but has been supported by theoretical considerations (Phillips 1977) and is used extensively in oceanography, where it predicts the interfacial stress in terms of the air speed at a reference height (usually 10 m) when coupled with the logarithmic velocity profile of aerodynamically rough, turbulent flows. Charnock's relation is

$$\epsilon = \frac{\gamma U_{G_\tau}^2}{g} \qquad [1]$$

where $U_{G_\tau}$ is the friction velocity $(\tau_i/\rho_G)^{1/2}$, $\tau_i$ is the interfacial stress, $\gamma$ is constant under fully developed conditions and $\epsilon$ is the roughness length. Sinai (1983a) combined a modified form of [1] with the classical relation between mean flow and roughness for fully rough internal flow (e.g. Streeter & Wiley 1979):

$$\frac{U_G}{U_{G_\tau}} \simeq 2.5 \ln\left(\frac{R}{\epsilon}\right) + 4.73. \qquad [2]$$

The result was

$$\hat{U}_G = -5\hat{U}_\tau \ln \hat{U}_\tau, \qquad [3]$$

where the circumflex denotes non-dimensional velocities, measured relative to $(20.1\ V^2/\xi)^{1/2}$. Here (see figure 1):

$$\xi = \frac{\eta C_i}{C_G + C_i}; \quad V^2 = gR_G \cos\theta; \quad R_G = \frac{2A_G}{C_i}. \qquad [4]$$

Comparison of [3] with Wallis & Dobson's (1973) experiments fixed the disposable parameter:

$$\eta \simeq \frac{545\rho_G}{\rho_L - \rho_G}; \qquad [5]$$

$\rho$ is the density, and the subscripts G and L denote properties appertaining to gas and liquid, respectively. Unfortunately, Wallis & Dobson did not report pressure gradients or interfacial stress, but their measurements are important because their rig offered a facility for adjusting the channel slope with the purpose of producing a constant time-mean liquid depth. Detailed calculations for two-dimensional turbulent flow (Sinai 1983b) allowed an approximate deduction of Wallis & Dobson's interfacial stress, and the agreement was encouraging except near slug or annular transition and under smooth conditions. The present contribution attempts to improve the model by incorporating smooth and transitional flows (*vis-à-vis* the roughness) as well as an empirical correction near slug transition.

## THEORY AND COMPARISON WITH EXPERIMENT

The first modification of the existing theory is effected by invoking the Colebrook–White formula relating skin friction to roughness, viscosity and mean flow for the complete range of smooth and rough turbulent flows (e.g. Streeter & Wylie 1979):

$$C_f^{-\frac{1}{2}} = -1.72 \ln\left(\frac{\epsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\, C_f^{\frac{1}{2}}}\right). \qquad [6]$$

Re is the channel Reynolds number $2RU/\nu$, where $\nu$ is the kinematic viscosity. This relation is applied to the gas phase, with $D$ interpreted as the mean hydraulic diameter for that phase. As before, $\hat{U}_G$ and $\hat{U}_\tau$ are defined as, cf. [4],

$$\hat{U}_G = \left(\frac{\xi}{20.1}\right)^{\frac{1}{2}} \frac{U_G}{V} \quad \text{and} \quad \hat{U}_\tau = \left(\frac{\xi}{20.1}\right)^{\frac{1}{2}} \frac{U_{G_\tau}}{V}; \qquad [7]$$

whereupon [6] becomes

$$\hat{U}_G = -2.5\, \hat{U}_\tau \ln\left(\hat{U}_\tau^2 + \frac{0.89}{\mathrm{Ga}\, \hat{U}_\tau}\right), \qquad [8a]$$

where Ga is a modified Galileo number:

$$\mathrm{Ga} = \frac{(gR_G^3 \cos\theta)^{\frac{1}{2}}}{\nu_G}. \qquad [8b]$$

Equation [8a] is now expected to yield better agreement with experiment when the interface is smooth, but it will still suffer from the poor predictions of $\hat{U}_\tau$ near transition (Sinai 1983b). On the basis of the latter paper, an empirical modification is being proposed as follows. Write [8a] as

$$\hat{U}_G = F(\hat{U}_\tau); F = -2.5\, \hat{U}_\tau \ln\left(\hat{U}_\tau^2 + \frac{0.89}{\mathrm{Ga}\, \hat{U}_\tau}\right). \qquad [9]$$

Then, formally,

$$\hat{U}_\tau = G(\hat{U}_G) \qquad [10]$$

where $G$ is the inverse function of $F$. Equation [10] is changed to

$$\hat{U}_\tau = H(\hat{U}_G) G(\hat{U}_G), \qquad [11]$$

where $H$ is an arbitrary function to be chosen in such a way that $\hat{U}_\tau$ is reduced only near transition without interfering with the condition:

$$\frac{d\hat{U}_G}{d\hat{U}_\tau} = 0; \text{transition.} \qquad [12]$$

The latter expresses the fact that as transition is approached the interfacial stress rises rapidly (e.g. Taitel *et al.* 1982; Sinai 1983a). It follows from [10] and [11] that

$$\hat{U}_G = F(\chi); \chi = \frac{\hat{U}_\tau}{H(\hat{U}_G)}. \qquad [13]$$

Therefore, if the subscript T denotes transition from the stratified to the slug or annular regimes, $\chi_T$ is defined by

$$F'(\chi_T) = 0, \qquad [14]$$

provided $(1 + \hat{U}_\tau H' F'/H^2)_T \neq 0$, where the prime denotes differentiation. Hence,

$$\frac{\ln\left(\chi_T^2 + \dfrac{a}{\chi_T}\right) + \left(2\chi_T^2 - \dfrac{a}{\chi_T}\right)}{\left(\chi_T^2 + \dfrac{a}{\chi_T}\right)} = 0, \qquad [15a]$$

where

$$a = \frac{0.89}{\text{Ga}}. \qquad [15b]$$

The form of $H$ being proposed tentatively is

$$H(\hat{U}_G) = 1 - \left(1 - \frac{\hat{U}_{\tau_T}}{\chi_T}\right)\left(\frac{\hat{U}_G}{\hat{U}_{G_T}}\right)^{0.7}. \qquad [16]$$

It would be helpful to emphasize that the previous, unmodified theory predicted

$$\hat{U}_{G_T} = 5\,e^{-1} \simeq 1.84 \qquad [17]$$

and

$$\hat{U}_{\tau_T} = e^{-1} \simeq 0.37. \qquad [18]$$

Equation [17] was found to be realistic (Sinai 1983a) when compared with Wallis & Dobson (1973), as well as Taitel & Dukler (1976) when the voidage is close to 0.5. However, [18] appears to overpredict $\hat{U}_{\tau_T}$ by a factor of about 2, and the values of that quantity deduced from the experiments of Wallis & Dobson (1973), Hanratty & Engen (1957) and Cohen & Hanratty (1968) lie close to 0.18–0.20.

The system of equations [13]–[15a, b] is complete, now that a fixed value of about 0.2 can be assigned to $\hat{U}_{\tau_T}$ (for any geometry), and the following procedure is recommended:

(i) solve equation [15a] for $\chi_T$;
(ii) determine $\hat{U}_{G_T}$ from [13]—Ga is usually large, and $\hat{U}_{G_T}$ will usually be close to the value given in [17];
(iii) specify $\hat{U}_{\tau_T}$—about 0.2;
(iv) [13] is now defined as an implicit relation between $\hat{U}_G$ and $\hat{U}_\tau$.

The results for a square channel (with zero mean liquid flow) are compared with Wallis & Dobson's measurements in figures 2 and 3 using the technique described by Sinai (1983b), which relates the channel slope to $\hat{U}_\tau$. That method is expected to become increasingly inaccurate as the voidage is increased, since it relies on a domination by the interface on the total pressure drop and on the length of the interface $C_i$ being a significant proportion of the total gas-phase perimeter $C_i + C_G$. Thus, the disagreement at $\alpha = 0.84$ may well be only apparent and spurious, and the comparisons for lower voidages are encouraging as they are.

Kim *et al.* (1985) suggest that the interfacial friction factor (defined by them as $\hat{C}_f = 2\tau_i/\rho_G(U_G - U_i)^2$, where $U_i$ is the interface velocity), is independent of the gas

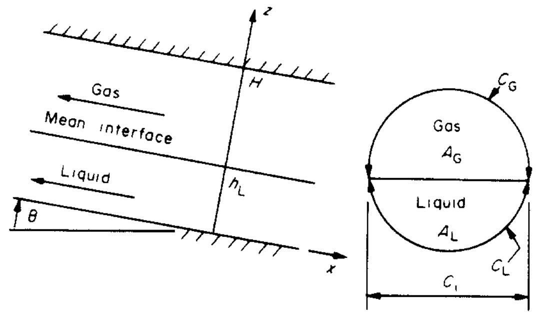


Figure 1. The configuration.

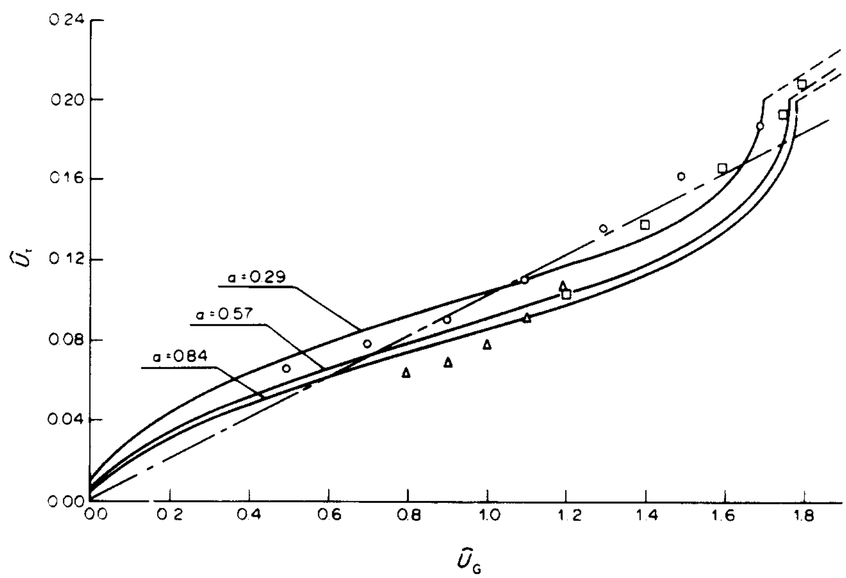


Figure 2. $\hat{U}_\tau$ vs $\hat{U}_G$: comparison of theory with experiment. Theory, ——. Experiment: ○, $\alpha = 0.29$; □, $\alpha = 0.57$; △, $\alpha = 0.84$. Kim *et al.* (1985), ---.

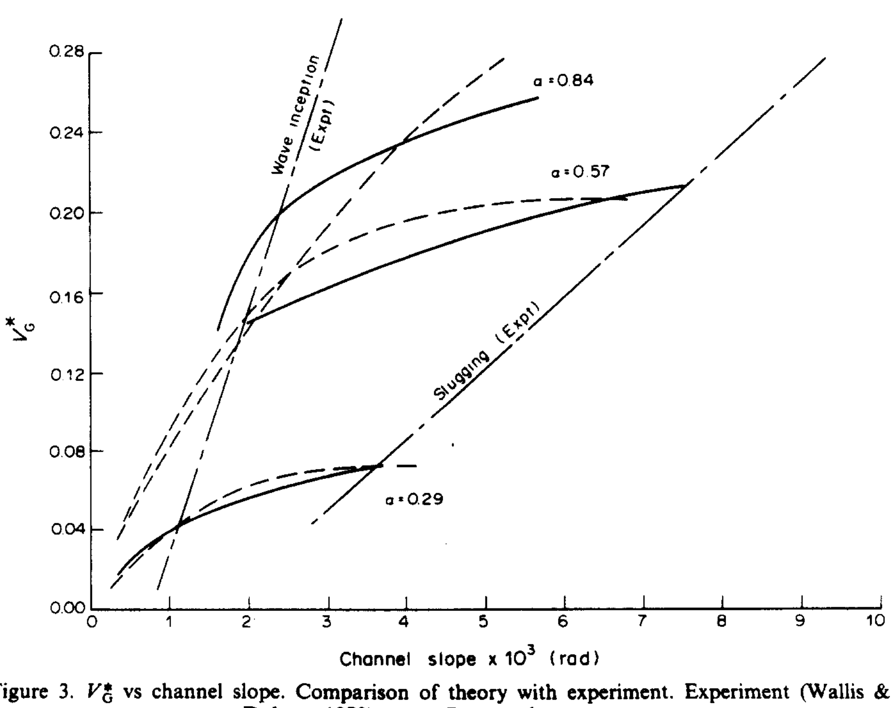


Figure 3. $V_G^*$ vs channel slope. Comparison of theory with experiment. Experiment (Wallis & Dobson 1973), ——. Present theory, ---.

Reynolds number for those conditions in which the three-dimensional wave regime exists on the water surface. Noting that $\hat{C}_f = 2\hat{U}_\tau^2/(\hat{U}_G - \hat{U}_i)^2$, lines of constant $\hat{C}_f$ in figure 2 are straight lines passing through the origin only if $U_i$ is negligible. Observe also, that $U_i$ is not exactly zero when $U_L$ vanishes, since recirculations are established in the liquid. Indeed, defining and correlating $\hat{C}_f$ in terms of $U_G - U_i$ is inconvenient for predictive purposes because $U_i$ is not easily calculated, particularly for conduits which are not rectangular. As it stands, [13] appears to refer only to the gas flow rate, but this is simply due to the underlying assumption that $|U_G| \gg |U_L|$. Two approaches are proposed in this paper when the latter condition is not satisfied. The first is simple but applies to general geometry and is expected to yield reasonable accuracy. It consists simply of [13] with $\hat{U}_G$ replaced by $\hat{U}_G - \hat{U}_L$ (Wallis & Dobson 1973):

$$\hat{U}_G - \hat{U}_L = F(\chi). \tag{19}$$

The second technique is more detailed in that it recognizes that the interfacial phenomena depend more directly on $U_i$ than on $U_L$ (even if the last two quantities are related). Equation [13] is replaced by the two simultaneous equations for the quantities $\hat{U}_\tau$ and $\hat{U}_i$:

$$\hat{U}_G - \hat{U}_i = F(\chi) \tag{20a}$$

and

$$2\hat{U}_\tau^2 = \hat{C}_f(\hat{U}_G - \hat{U}_i)^2, \tag{20b}$$

where

$$\chi = \frac{\hat{U}_\tau}{H(\hat{U}_G - \hat{U}_i)} \tag{21}$$

and $\hat{C}_f$ is the coefficient correlated by Kim *et al.* However, care must be taken to apply [20a, b] only to flows and geometries encompassed by Kim *et al.*'s correlations, or by any future correlations which may be developed. Equation [20a, b] represents a determinate system, whereas $\hat{C}_f$ in isolation is insufficient for determining $\tau_i$ unless $U_i$ is negligible.

Kim *et al*'s correlation is shown in figure 2, for zero liquid Reynolds number and interfacial velocity; over certain ranges it lies close to the present theory.

It only remains to discuss the method when the voidage is sufficiently high to preclude transition to slugs. In such circumstances, the flow approaches the annular regime as the gas flow increases, and it is proposed here that for $\hat{U}_G > \hat{U}_{G_T}$, a linear variation is assumed between the point $\hat{U}_{G_T}$ and $\hat{U}_{\tau_T}$ and the point prediction by the triangular relationship of annular flow (e.g. Butterworth & Hewitt 1979), for the same liquid superficial velocity. Indeed, the method proposed by Sinai (1983b, 1986) may itself be described as a triangular relationship for stratified flow, since the interfacial relationship links the liquid and gas volumetric flows.

## CONCLUSIONS

On the basis of a "deductive" comparison (Sinai 1983b) of Sinai's original proposal (1983a) for a model of interfacial stress with Wallis & Dobson's (1973) measurements, this paper presents a generalization to span the complete range of interfacial roughness as well as an empirical correction aimed at improving the prediction of the interfacial stress as transition is approached. If slug transition does not occur, this paper simply proposes a linear variation on the $\hat{U}_G$, $\hat{U}_\tau$ plane between $\hat{U}_{G_T}$, $\hat{U}_{\tau_T}$ and the stratified annular transition point described by the triangular relationship of annular flow. The modified theory compares favourably with Wallis & Dobson's measurements at low voidages, but there is some uncertainty about the validity of the comparison at high voidages.

Two techniques are proposed for accounting for the liquid's movement. The first only considers the mean liquid velocity $\hat{U}_L$, and only requires one algebraic equation [19] to be solved. The second regards the time-mean interfacial velocity as an unknown too, in addition to the interfacial stress, and involves the simultaneous solution of two equations [20a, b].

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